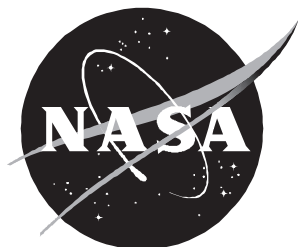


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# A Simple Method for Nucleon-Nucleon Cross Sections in a Nucleus

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April 1999

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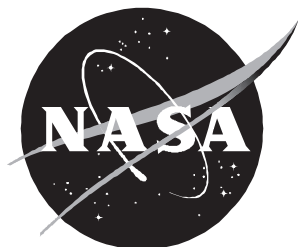
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# Abstract

*A simple reliable formalism is presented for obtaining nucleon-nucleon cross sections within a nucleus in nuclear collisions for a given projectile and target nucleus combination at a given energy for use in transport, Monte Carlo, and other calculations. The method relies on extraction of these values from experiments and has been tested and found to give excellent results.*

## 1. Introduction

A significant amount of information about nucleon-nucleon (NN) collisions is obtained from experiments performed for free NN collisions. This information, when used for nucleon-nucleus and nucleus-nucleus collisions, has to be modified, mainly because of the presence of other nucleons in a nucleus (ref. 1). However, no simple reliable method obtained directly from experiments exists for this modification. The present work fills this void and gives reliable values of nucleon-nucleon cross sections in a nucleus for a given projectile target system at a given energy. This method will also be useful for transport and Monte Carlo calculations.

## 2. Method

Following the coupled channel approach (refs. 2 to 7) developed at the Langley Research Center, recently we have extracted the values of nucleon-nucleon cross sections in a nucleus medium from experimental absorption cross sections. (See refs. 8 and 9.) The detailed steps and expressions used for this extraction are given in references 8 and 9; however, we sketch here some of the steps to make the present method more clear.

The absorption method in this formalism is given by

$$\sigma_{\text{abs}} = 2\pi \int_0^\infty b \, db \{1 - \exp[-2 \operatorname{Im}(\chi)]\} \quad (1)$$

where  $b$  is the projectile impact parameter and  $\chi$ , the eikonal phase matrix (see refs. 2 to 7 for details), is given by

$$\chi(\mathbf{b}) = \chi_{\text{dir}}(\mathbf{b}) - \chi_{\text{ex}}(\mathbf{b}) \quad (2)$$

The direct and exchange terms are calculated by using the following expressions (refs. 2 to 5):

$$\chi_{\text{dir}}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}} \int d^2 \mathbf{q} \times \exp(i\mathbf{q} \cdot \mathbf{b}) F^1(-\mathbf{q}) G^1(\mathbf{q}) f_{NN}(\mathbf{q}) \quad (3)$$

and

$$\chi_{\text{ex}}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}} \int d^2 \mathbf{q} \exp(i\mathbf{q} \cdot \mathbf{b}) F^1(-\mathbf{q}) G^1(\mathbf{q}) \times \frac{1}{(2\pi)^2} \int d^2 \mathbf{q}' \exp(i\mathbf{q}' \cdot \mathbf{b}) f_{NN}(\mathbf{q} + \mathbf{q}') C(\mathbf{q}') \quad (4)$$

where  $F^1$  and  $G^1$  are projectile and target ground-state one-body form factors, respectively, and  $C$  is the correlation function (ref. 5). The mass numbers of projectile and target nuclei are represented by  $A_P$  and  $A_T$ , respectively. The two-body amplitude  $f_{NN}$  is parameterized as

$$f_{NN} = \frac{\sigma(\alpha + i)}{4\pi} k_{NN} \exp\left(-\frac{Bq^2}{2}\right) \quad (5)$$

where  $k_{NN}$  is the relative wave number in the two-body center of mass system,  $\sigma$  is the two-body cross section,  $B$  is the slope parameter, and  $\alpha$  is the ratio of the real part to the imaginary part of the forward two-body amplitude.

Notice that the absorption cross section in equation (1) depends on the imaginary part of the eikonal phase matrix. Then the two-body amplitude in the medium  $f_{NN,m}$  can be written as

$$f_{NN,m} = f_m f_{NN} \quad (6)$$

where  $f_{NN}$  is the free NN amplitude and  $f_m$  is the system and energy dependent medium multiplier function (refs. 8 and 9). The nucleon-nucleon cross sections in the medium ( $\sigma_m$ ) can then be written as

$$\sigma_{NN,m} = f_m \sigma_{NN} \quad (7)$$

where  $\sigma_{NN}$  is the nucleon-nucleon cross section in free space and the medium multiplier is given by

$$f_m = 0.1 \exp\left(-\frac{E}{12}\right) + \left[1 - \left(\frac{\rho_{av}}{0.14}\right)^{1/3} \exp\left(-\frac{E}{D}\right)\right] \quad (8)$$

where, for  $A_T < 56$ ,

$$D = 46.72 + 2.21 A_T - (2.25 \times 10^{-2}) A_T^2 \quad (9)$$

and for  $A_T \geq 56$ ,

$$D = 100 \quad (10)$$

In equation (8),  $\rho_{av}$  refers to the average density of the colliding system and is given by

$$\rho_{av} = \frac{1}{2}(\rho_{A_P} + \rho_{A_T}) \quad (11)$$

where the density of a nucleus  $A_i$  ( $i = P, T$ ) is calculated in the hard sphere model and is given by

$$\rho_{A_i} = \frac{A_i}{\frac{4\pi}{3} r_i^3} \quad (12)$$

where the radius of the nucleus  $r_i$  is defined by

$$r_i = 1.29 (r_i)_{rms} \quad (13)$$

The root-mean-square radius  $(r_i)_{rms}$  is obtained directly from experiment (ref. 10) after “subtraction” of the nucleon charge form factor (ref. 2).

Use of equations (7) to (10) modifies free neutron-proton cross sections ( $\sigma_{np}$ ) and proton-proton cross sections ( $\sigma_{pp}$ ), which in turn can be used to calculate nucleon-nucleon (NN) cross sections for any system

of colliding nuclei by taking the isospin average value as

$$\sigma_{NN} = \frac{Z_P Z_T + N_P N_T}{A_P A_T} \sigma_{pp} + \frac{N_P Z_T + Z_P N_T}{A_P A_T} \sigma_{np} \quad (14)$$

where  $Z_P$ ,  $N_P$ , and  $A_P$  ( $Z_T$ ,  $N_T$ , and  $A_T$ ) are the charge, neutron, and mass number of the projectile (target) nucleus.

### 3. Procedure

For convenience, we fit free neutron-proton (np) and proton-proton (pp) cross section data (refs. 11 to 13) at available energies by following smooth functions of energy for ease of interpolation and for their availability at a given energy:

$$\begin{aligned} \sigma = & A \exp\left(-\frac{E}{25}\right) + \frac{B}{1 + \exp[(400 - E)/75]} \\ & + \frac{C}{1 + \exp[(E - 300)/75]} \\ & + 28 \exp\left(-\frac{E}{125}\right) \cos(26E^{0.453}) \end{aligned} \quad (15)$$

where for pp system,  $A = 300$ ,  $B = 44$ , and  $C = 15$  and for np systems,  $A = 922$ ,  $B = 40$ , and  $C = 50$ . Figures 1 and 2 show the present representation (eq. (15)) of free np and pp cross sections compared with the available experimental data (refs. 11 to 13).

The procedure for obtaining in-medium cross sections is as follows. For a given projectile colliding with a target at laboratory energy ( $A$  MeV), the medium multipliers are calculated by using equations (8) to (13). Having calculated the medium multiplier, equation (7) is used to calculate in-medium np and pp cross sections, and then in-medium NN cross sections are calculated by using equation (14).

### 4. Discussion and Results

Figures 3 to 6 show results of in-medium cross sections in proton-nucleus collisions for a set of representative nuclei. For any other particular case, our formalism can be used to get specific values. Note that at lower energies (less than 200 A MeV), there is system

dependence of the cross sections, and at higher energies between 200 A MeV and 600 A MeV the system dependence is less important. It is also interesting to note that for energies greater than 600 A MeV, the medium modifications become less important and the free nucleon-nucleon cross sections appear adequate.

Figures 7 to 10 give medium modified cross sections for neutron-nucleus collisions for various systems. For comparison with the proton-nucleus systems, we have kept the same set of nuclei for these collisions. The general features of the proton-nucleus collisions are seen in the neutron-nucleus collisions as well. However, it is very interesting to compare the modified cross sections for the two cases. We note that the modified cross sections needed for the proton-nucleus system are very different from the neutron-nucleus systems. This observation is interesting, and to our knowledge has not been pointed out in the literature before; it will significantly impact the applications.

Figures 11 to 14 give results for nucleus-nucleus collisions; to make our points, we have chosen carbon as the projectile. However, we need to emphasize again that for a specific case of interest it will be good

to generate system modified cross sections by using our formalism rather than taking the values from other systems because the values are system dependent as shown; this is a really interesting observation. For nucleus-nucleus collisions, the system modified cross sections are significantly different from the proton-nucleus and neutron-nucleus collisions. Our calculation seems to indicate that not only the nuclear force but the Coulomb force also plays a role in modifying the cross sections in a nucleus; this effect has been ignored by others.

## 5. Concluding Remarks

Very interesting features have been shown for the system dependence of the cross sections for various systems. The system dependent cross sections depend both on the kinetics and the dynamics of the system. These effects are important. Our formalism includes them in a simple and reliable way. In addition, our approach points out several interesting features which were not recognized by others. Our method is easy to use and gives excellent agreement with experiments. The usefulness of our method for various applications—transport, Monte Carlo, radiation protection, radiation therapy—is noted.

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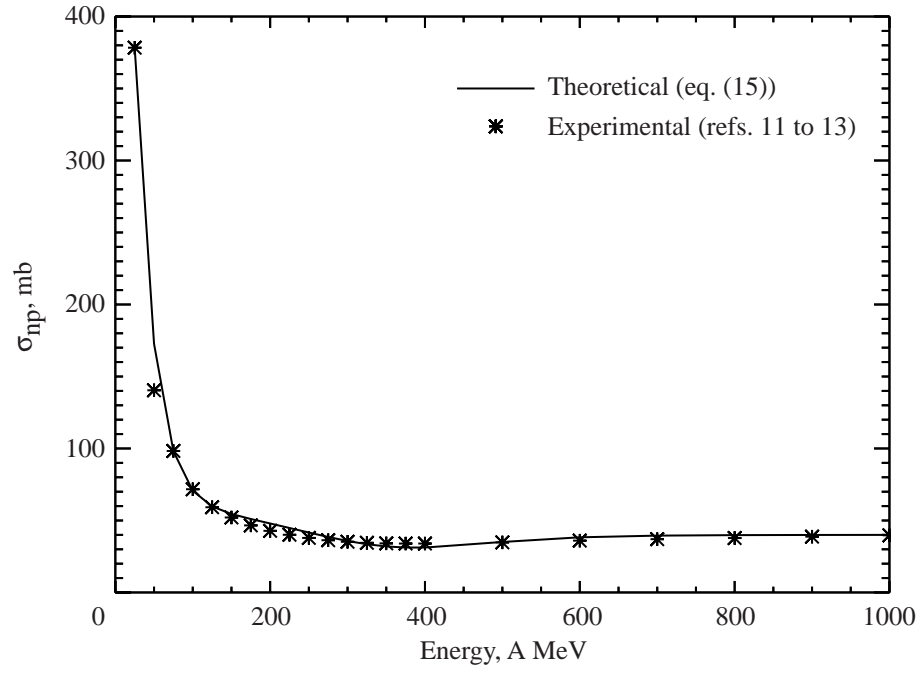


Figure 1. Free neutron-proton cross sections  $\sigma_{np}$ .

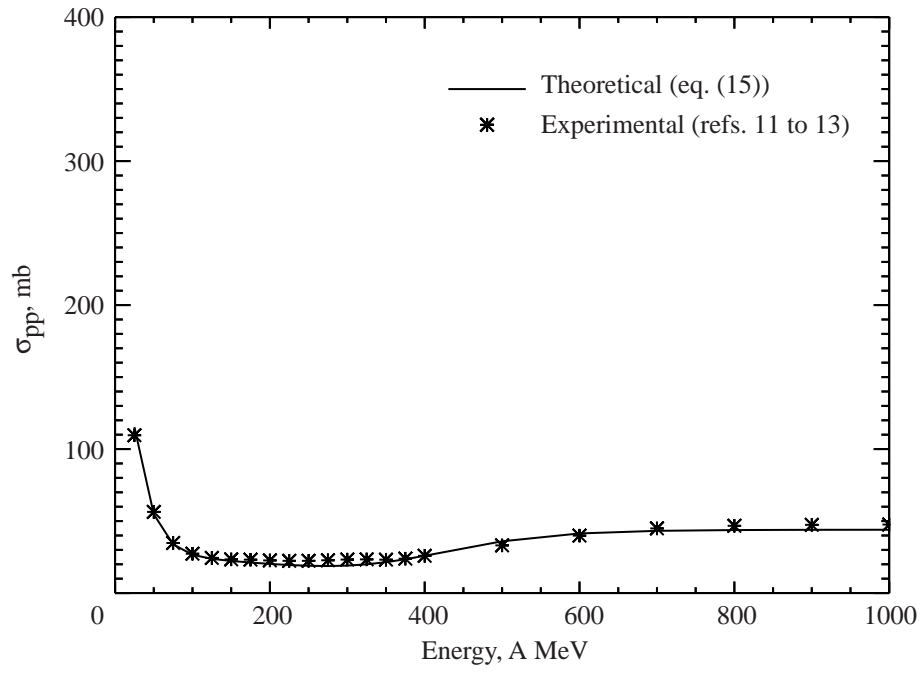


Figure 2. Free proton-proton cross sections  $\sigma_{pp}$ .

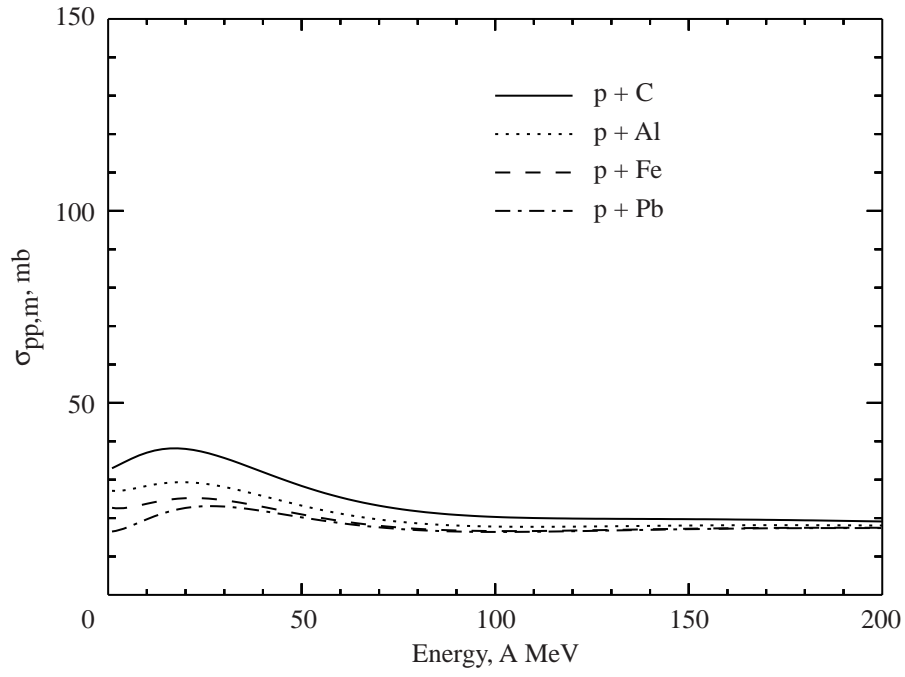


Figure 3. Modified proton-proton cross sections  $\sigma_{pp,m}$  for various proton-nucleus collisions.

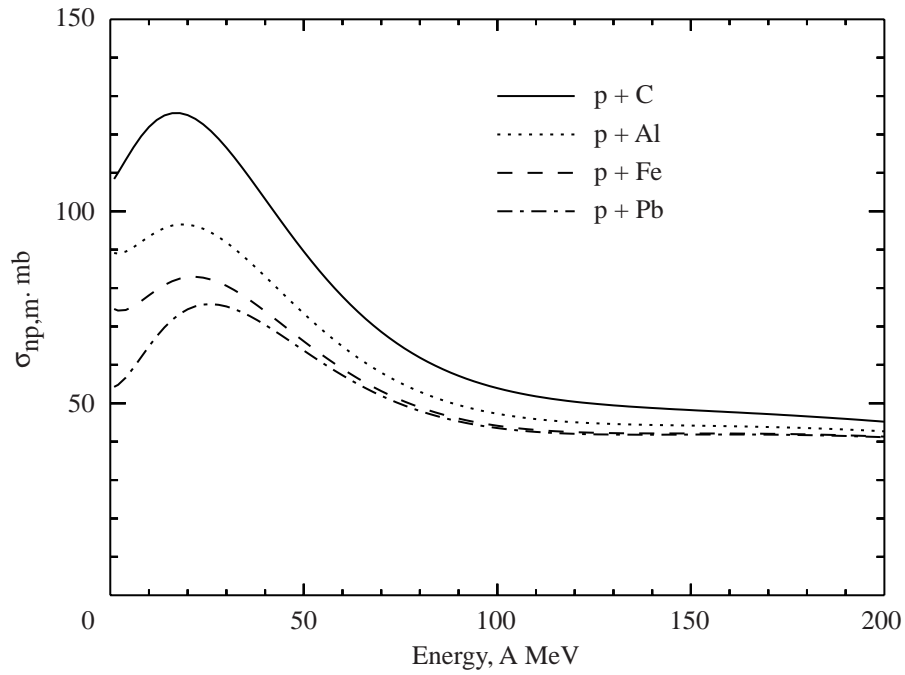


Figure 4. Modified neutron-proton cross sections  $\sigma_{np,m}$  for various proton-nucleus collisions.

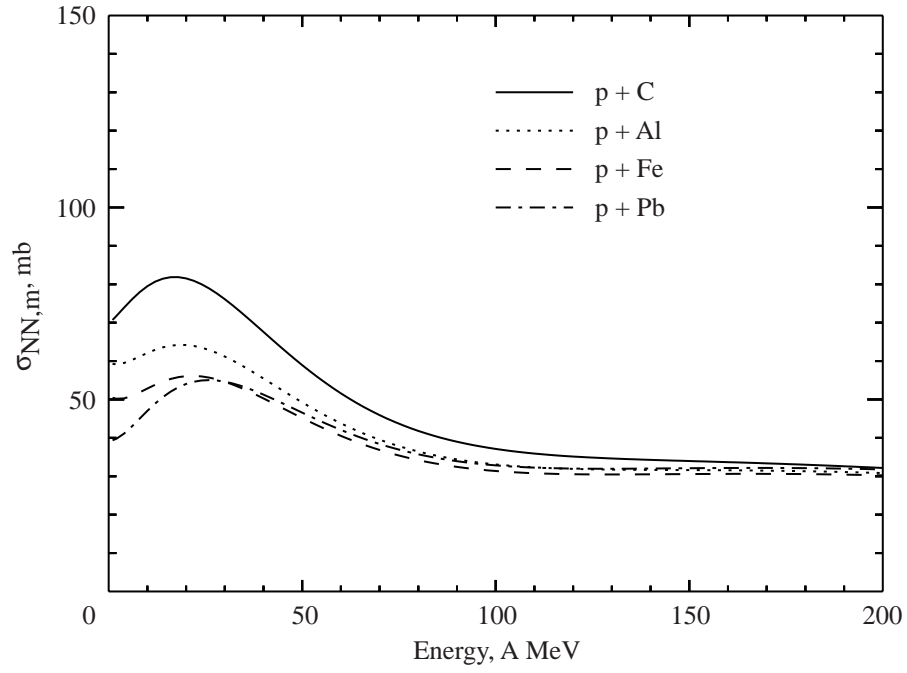


Figure 5. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various proton-nucleus collisions.

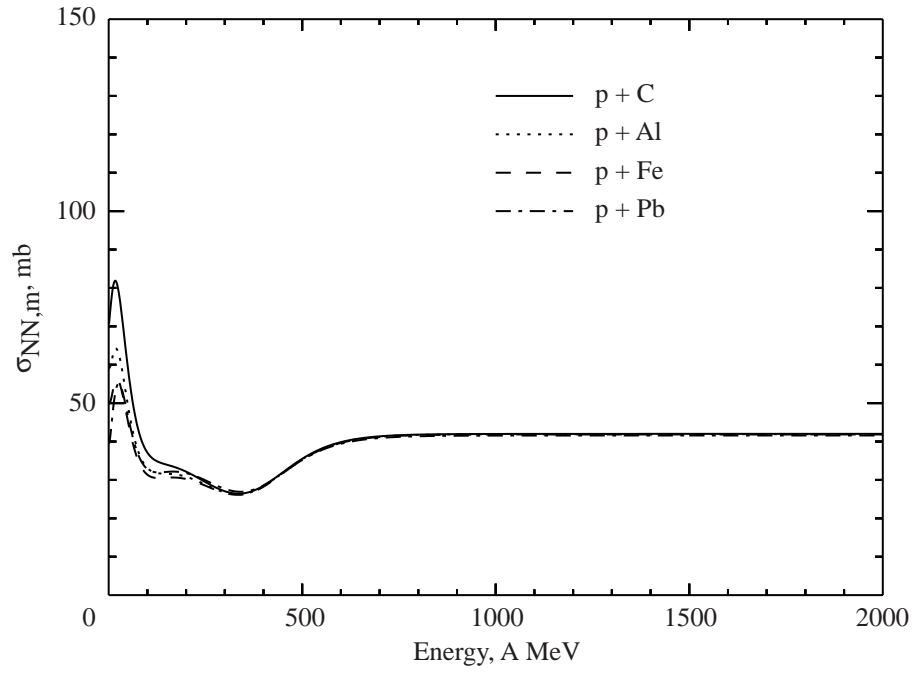


Figure 6. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various proton-nucleus collisions with energy range extended to 2 A GeV to demonstrate variation of nucleon-nucleon cross sections with energy.

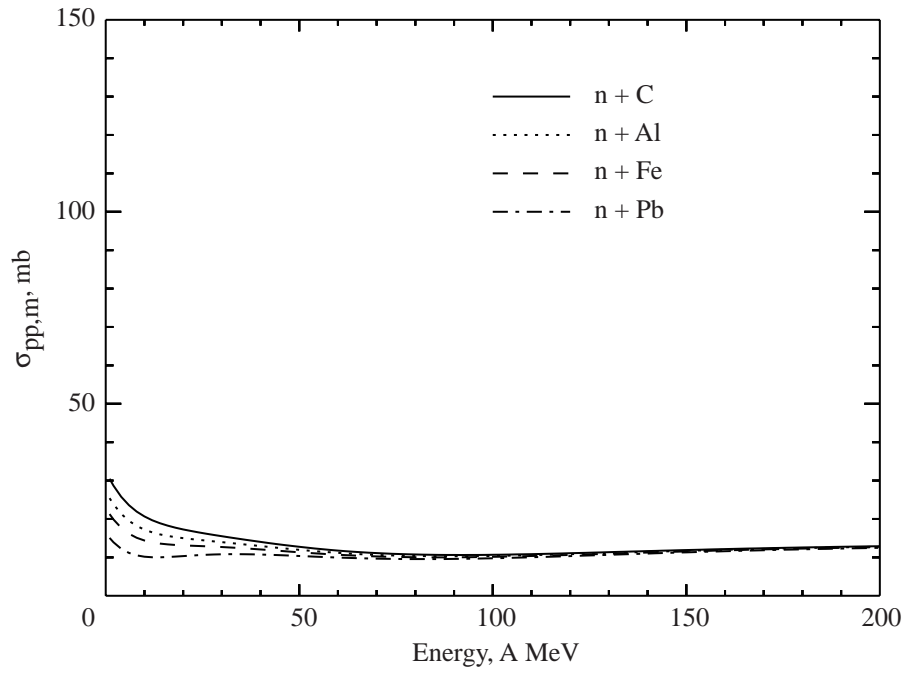


Figure 7. Modified proton-proton cross sections  $\sigma_{pp,m}$  for various neutron-nucleus collisions.

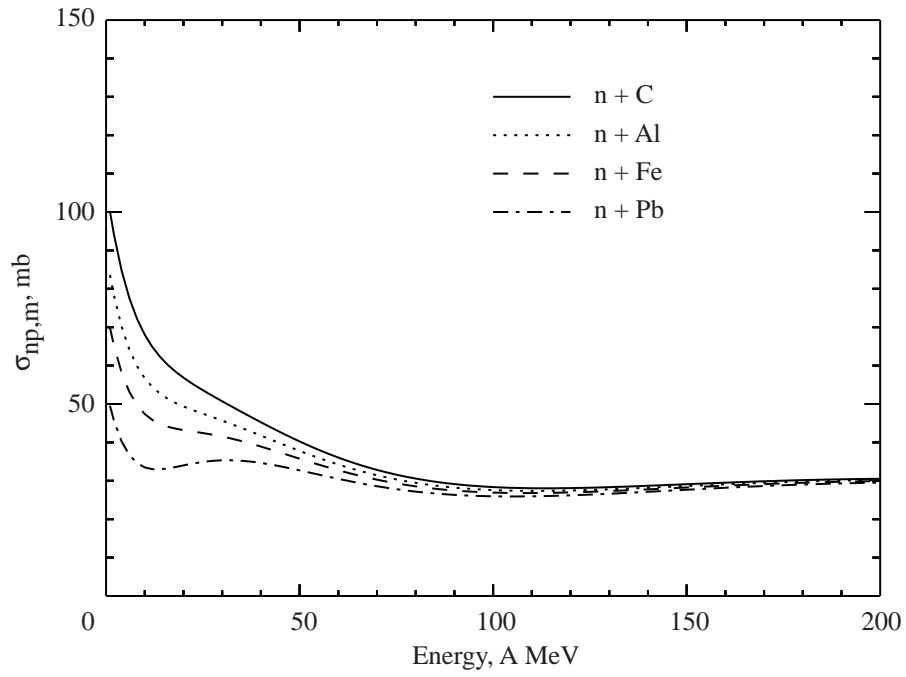


Figure 8. Modified neutron-proton cross sections  $\sigma_{np,m}$  for various neutron-nucleus collisions.

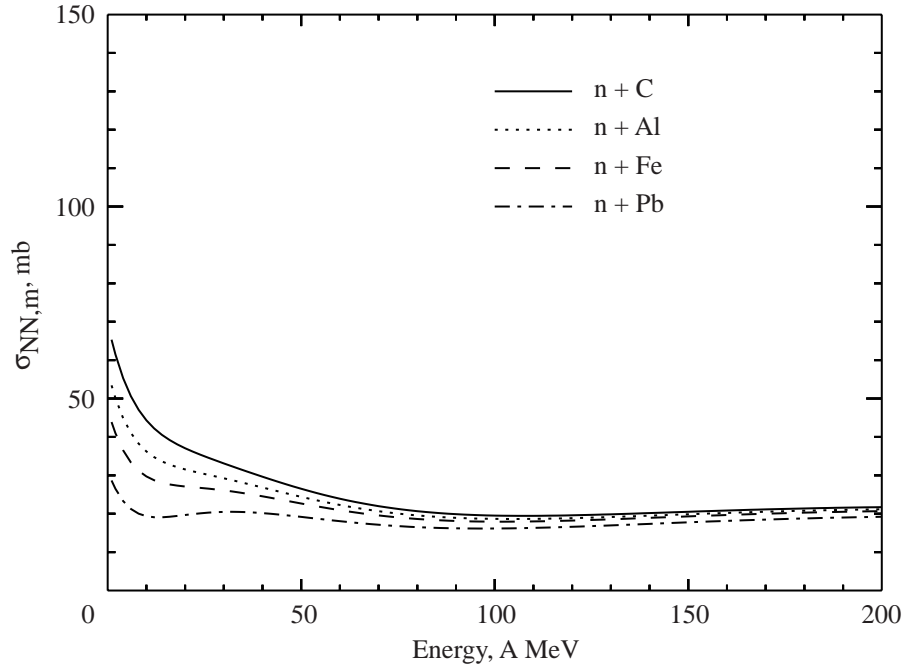


Figure 9. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various neutron-nucleus collisions.

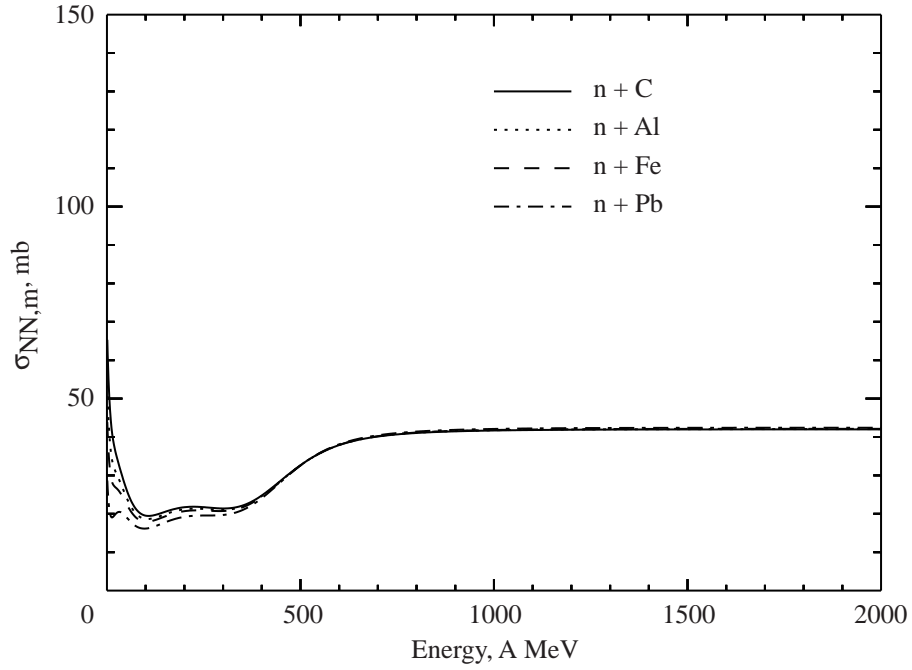


Figure 10. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various neutron-nucleus collisions with energy range extended to 2 A GeV to demonstrate variation of nucleon-nucleon cross sections with energy.

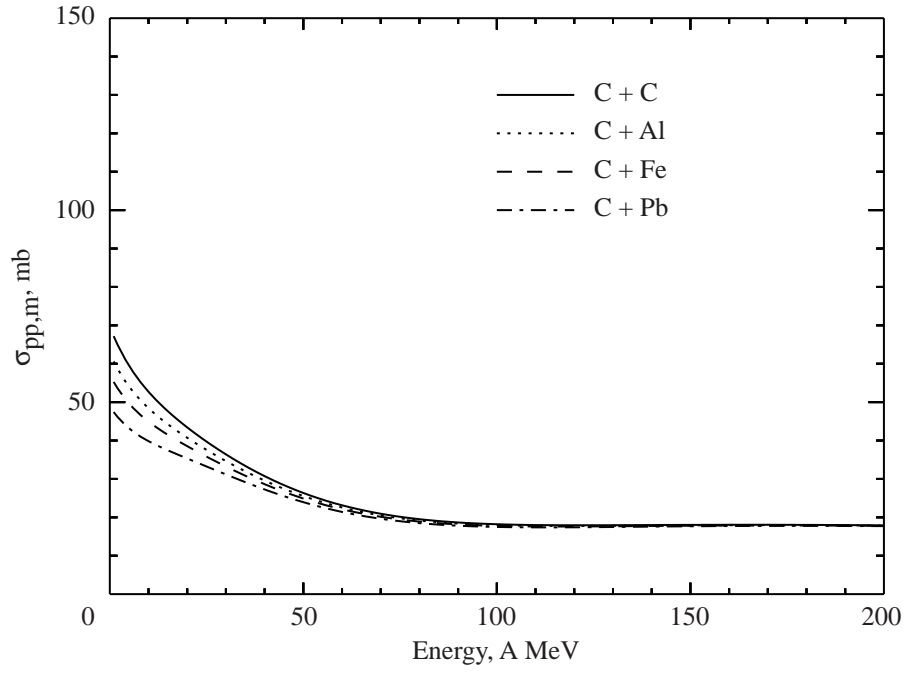


Figure 11. Modified proton-proton cross sections  $\sigma_{pp,m}$  for various nucleus-nucleus collisions.

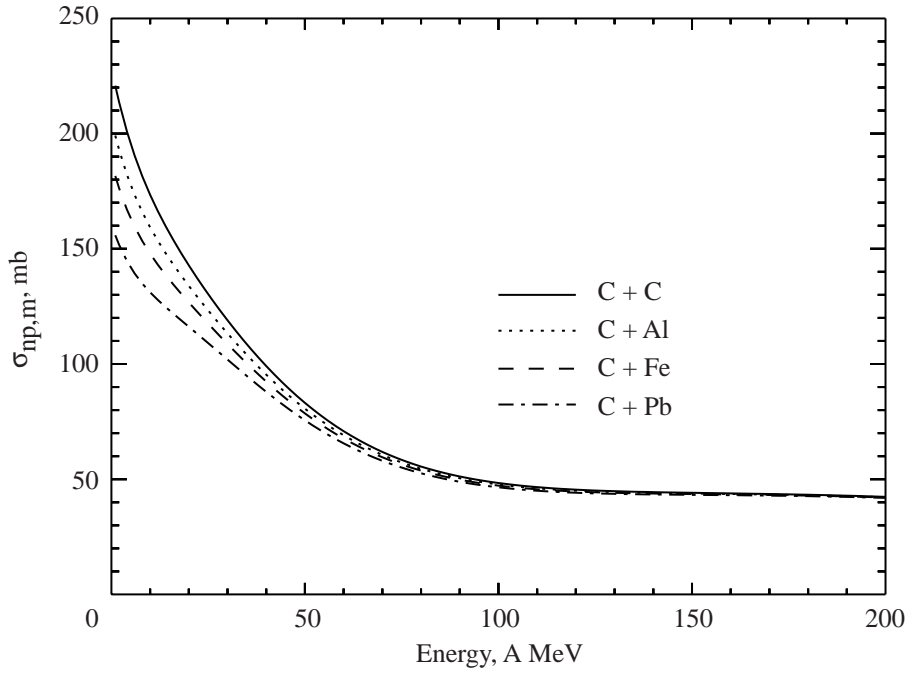


Figure 12. Modified neutron-proton cross sections  $\sigma_{np,m}$  for various nucleus-nucleus collisions.

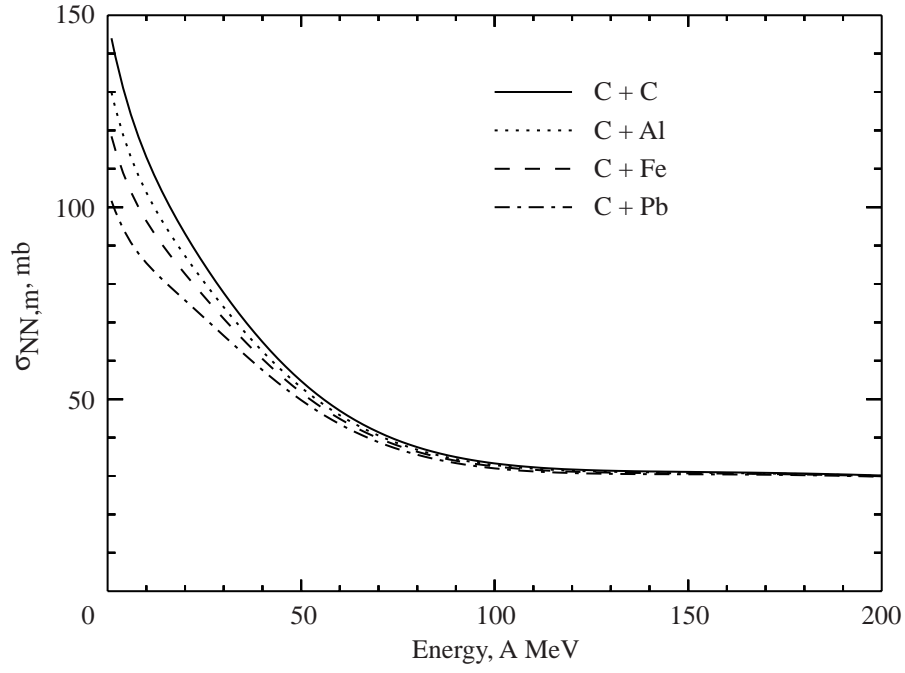


Figure 13. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various nucleus-nucleus collisions.

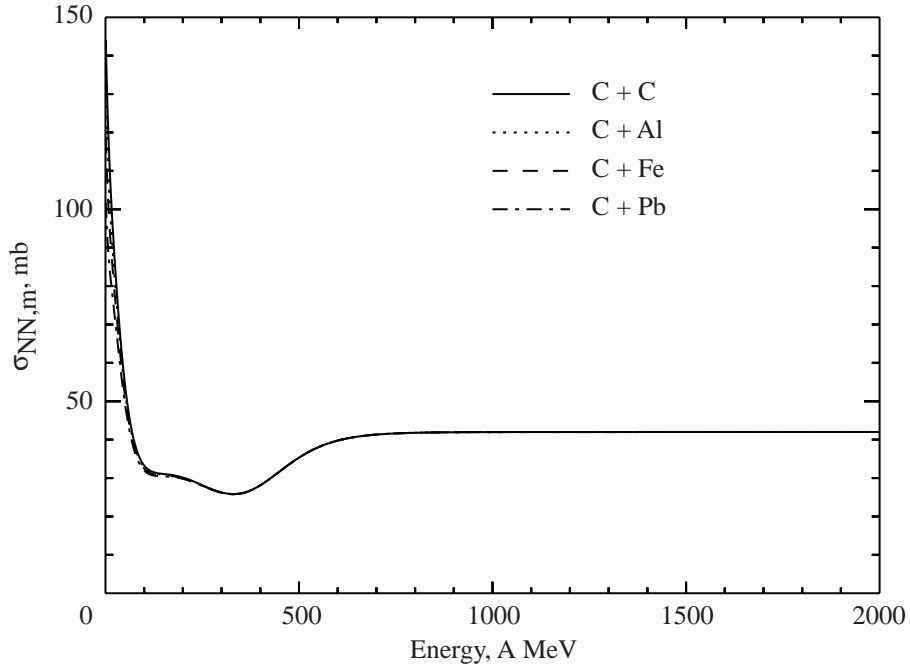


Figure 14. Modified nucleon-nucleon cross sections  $\sigma_{NN,m}$  for various nucleus-nucleus collisions with energy range extended to 2 A GeV to demonstrate variation of nucleon-nucleon cross sections with energy.

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